Slope safety evaluation by integrating multi-source monitoring information

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Abstract
A systematic method is presented for evaluating the slope safety utilizing multi-source monitoring information. First, a Bayesian network with continuously distributed variables for a slope involving the factor of safety, multiple monitoring indexes and their influencing parameters (e.g. friction angle and cohesion) is constructed. Then the prior probabilities for the Bayesian network are quantified considering model and parameter uncertainties. After that, multi-source monitoring information is used to update the probability distributions of the soil or rock model parameters and the factor of safety using Markov chain Monte Carlo simulation. An example of a slope with multiple monitoring parameters is presented to illustrate the proposed methodology. The method is able to integrate multi-source information based on slope stability mechanisms, and update the soil or rock parameters, the slope factor of safety, and the failure probability with the integrated monitoring information. Hence the evaluation becomes more reliable with the support of multiple sources of site-specific information.

1. Introduction
Field monitoring is an important means to evaluate the safety state of slopes, provide basis for slope safety control measures, warn of impending failures and mitigate risks of slope failures [1,2]. Common instruments and measuring indexes in slope monitoring are summarized in Table 1 according to Marr [2]. With appropriate instruments, changes in slope characteristics such as soil stresses, pore water pressures and crack development can be measured and used as a basis for evaluating the slope safety.

Einstein and Sousa [1] emphasized that monitoring results and observations need to be properly interpreted to evaluate slope safety. Numerous studies have been conducted on the interpretation of slope safety based on monitoring information. Some studies aimed to find a relationship between rainfall and slope failure with statistical analysis [3–6] or mechanical analysis [7,8]. It is also believed that slope displacements, especially displacement incremental rates, are important indicators to evaluate slope safety [9,10]. Some advanced methods such as GIS and ground-based radar have been attempted for slope safety evaluation and warning [11–13].

Existing interpretation methods typically use one single index (e.g. surface or underground deformation, pore pressure, or rainfall) as a predictor and hence reveal only one aspect of slope performance. A holistic assessment of the slope safety state may not be achieved. Besides, a large portion of monitoring data is often not utilized in these methods. Slope safety evaluation using multiple sources of monitoring information may be more reasonable, and is the topic of the present research.

Data fusion is a process of integrating multiple sources of data and knowledge representing the same structure into a consistent, accurate and useful representation [14]. Chang et al. [15] and Wong et al. [16] conducted landslide hazard assessment based on the fusion of multisource remote sensing images of the same site. To reduce the errors of monitoring information from human mistakes or faulty equipment, Guo et al. [17] and Peng et al. [18] applied a data-fusion method to filter and extract monitoring information from multiple sensors. Available data fusion methods can improve the quality of monitoring data of the same type (e.g. displacement) by fusing the information from multiple sensors. However, information from different types of monitoring data (e.g. displacement, stress and pore water pressure) may not be integrated easily and applied to evaluate the slope safety in these methods. Artificial neural networks were sometimes used for landslide susceptibility assessment with multiple pieces of information of the influential factors (e.g. slope geometry, soil thickness and drainage) [19]. However, little physical analysis is involved in artificial neural networks; the uncertainties involved are not included explicitly either.

In this study, a systematic method of slope safety evaluation is presented utilizing multi-source monitoring information with Bayesian networks. The method is aimed to (1) integrate multi-
source information based on slope stability mechanisms; (2) update the soil or rock model parameters with the integrated monitoring information; and (3) predict the factor of safety and failure probability of the slope with the integrated monitoring information. An example of a slope with multiple monitoring indexes is presented to illustrate the proposed methodology.

2. Proposed method of slope safety evaluation by integrating multi-source monitoring information

2.1. The Bayesian network

A Bayesian network is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph [20]. A Bayesian network combines the knowledge of graph theory and statistics theory. It consists of nodes (parameters) and arcs/links (inter-relationships) with their (conditional) probabilities, which can be applied to solve uncertainty problems by logic reasoning. Bayesian networks have been proven to be a robust method for reliability analysis and risk analysis, including the integration of multiple sources of information [21–26].

2.2. The framework

Both the factor of safety, \( F_S \), and monitoring information are indicators of the performance of a slope of certain parameters (e.g. cohesion, friction angle etc.), as shown in Fig. 1. A change in the slope parameters will lead to changes in the monitoring information and slope safety. Based on this, the slope safety evaluation with multi-source monitoring information is conducted by updating the slope parameters using the monitoring information (back analysis), and calculating the \( F_S \) and the failure probability \( (F_F) \) of the slope with the updated slope parameters (inference) as shown in Fig. 1. The main steps of the proposed method are shown in Fig. 2:

- (1) A causal network is first constructed considering relationships among \( F_S \), monitoring parameters \( (M) \) and slope parameters \( (S) \).
- (2) The \( F_S \) and \( M \) as response functions of \( S \), namely, \( F_S(S) \) and \( M(S) \) in Fig. 2, are obtained using finite element analysis and a response surface method.
- (3) The prior probabilities of the Bayesian network, namely, the prior probabilities of \( S \), \( P(S) \), and the prior conditional probabilities of \( M \) given \( S \), \( P(M|S) \), and \( F_S \) given \( S \), \( P(F_S|S) \), are obtained by applying Monte Carlo simulation to the response functions, \( F_S(S) \) and \( M(S) \), as shown in Fig. 2. The model uncertainties are included in this step.
- (4) The posterior probabilities of \( S \), \( P(S|m) \), are obtained by updating the prior, \( P(S) \), with available monitoring information, \( M = m \), as shown in Fig. 2. Markov Chain Monte Carlo (MCMC) simulation is applied in this step since the Bayesian network in this study involves non-normal continuous probability distributions.
- (5) Finally, the posterior probability of \( F_S \), \( P(F_S|m) \), is calculated with the updated slope parameters as well as the probability of slope failure (Fig. 2).

Actually, the updating of the slope parameters and the updating of the factor of safety, namely, steps (4) and (5), are executed at the same time in the computation program. They are described in two steps here in order to better understand the updating mechanisms. The framework of the proposed method will be introduced step by step later in the paper. An example of a slope with multiple monitoring indexes will be presented to illustrate the proposed framework toward the end of this paper.

2.3. Constructing a Bayesian network by considering \( F_S \) and monitoring parameters

As shown in Fig. 3, a simple casual network is built for a soil slope by considering the soil parameters (i.e. cohesion \( c \) and friction angle \( \phi \)), two monitoring parameters (i.e. the vertical displacement at point \( A \), \( D_A \) and the vertical stress at point \( B \), \( S_B \)), \( F_S \) and the slope safety state (safe/failed or \( S/F \)). In this case, \( F_S \), \( D_A \) and \( S_B \) are governed by \( c \) and \( \phi \) if other parameters are well defined (\( c \) and \( \phi \) are the parents of \( F_S \), \( D_A \) and \( S_B \) ); \( S/F \) depends on the value of \( F_S \) (\( F_S \) is the parent of \( S/F \)), which means \( S/F = \text{safe} \) if \( F_S > 1 \) and \( S/F = \text{failed} \) if \( F_S < 1 \). \( c \) and \( \phi \) are uncertain and difficult to obtain, while \( D_A \) and \( S_B \) can be measured using monitoring instruments. Therefore in the Bayesian network, the monitoring information of \( D_A \) and \( S_B \) can be used to update \( c \), \( \phi \), \( F_S \) and \( S/F \).
The first is the numerical model error, which is produced in numerical simulation:
\[ g(c, \phi) = N(c, \phi) + \epsilon_1 \]  
where \( N(c, \phi) \) is the numerical result and \( \epsilon_1 \) is the numerical model error. The numerical model errors for different monitoring parameters may be correlated through the slope parameters.

The second is the response surface model error, which occurs in the response surface method:
\[ N(c, \phi) = R(c, \phi) + \epsilon_2 \]  
where \( R(c, \phi) \) is the simulated result from the response surface and \( \epsilon_2 \) is the response surface model error. Therefore, the real value can be expressed as
\[ g(c, \phi) = R(c, \phi) + \epsilon_1 + \epsilon_2 \]  

With Eq. (4), Monte Carlo simulation can be applied to obtain the conditional probabilities of \( F_S, D_A \) and \( S_F \) given \( c \) and \( \phi \).

2.6. Updating slope parameters, \( F_S \) and \( F/S \) with Markov chain Monte Carlo simulation

The slope safety evaluation using multi-source monitoring information is conducted by updating the slope parameters, \( F_S \) and \( S/F \) with site-specific monitoring information. In the Bayesian network as shown in Fig. 3, a target vector of variables, \( y \) [i.e. \( (D_A, S_F) \)] is expressed as
\[ y = g(\theta) = g(c, \phi) \]  
where \( \theta \) is a vector of random parameters, which is \( (c, \phi) \) in this case. Based on Bayes’ theorem \([30,31]\), the posterior probability density function of \( \theta \) given the monitoring information can be expressed as
\[ f(\theta | y) = kL(\theta | y) f(\theta) \]  
where \( k \) is a constant to make the probability density function valid, \( f(\theta) \) is the prior probability density function of \( \theta \) and \( L(\cdot | \cdot) \) is a likelihood function. If multi-source monitoring information is used [i.e. \( y = (D_A, S_F) \)], the pieces of observed information may be assumed independent, i.e. \( L(\theta | y) = L(\theta | D_A) \cdot L(\theta | S_F) \). With the updated \( \theta \), the posterior distribution of \( F_S \) can be predicted as
\[ f(F_S | y) = \int f(F_S | \theta) f(\theta | y) d\theta \]  
where \( f(F_S | \theta) \) is the distribution of \( F_S \) with parameters \( \theta \). The mean and variance of \( F_S \) are updated as
\[ \mu(F_S) = \int F_S f(F_S | y) dF_S \]  
\[ V(F_S) = \int (F_S - \mu(F_S))^2 f(F_S | y) dF_S \]

The probability of slope failure is given by
\[ P(S/F = \text{failed}) = P(F_S < 1) = \int_0^1 f(F_S | y) dF_S \]  
A key part of the above analysis is to update parameters \( \theta \) based on the monitoring information as shown in Eq. (6). Since \( f(\theta) \) and \( L(\theta | y) \) could be any types of continuous distributions, \( f(\theta | y) \) may not be obtained with conjugate distributions \([30]\). In this study, Markov Chain Monte Carlo (MCMC) simulation is introduced to solve this problem. In the MCMC method, one can use the previous sample values to randomly generate the next sample value, forming a Markov chain. The sample data in the Markov chain are
proved to follow the required distribution as shown in Eq. (6) [32]. Then the sampled data are used to obtain the estimator with Monte Carlo simulation according to Eqs. (7)-(10).

The Metropolis algorithm [32], which is one of the most popular ways to generate Markov chains, is adopted in this study to conduct the MCMC simulation. The Metropolis algorithm can be written as follows [32,33]:

1. Start with any initial value \( \theta_0 \) satisfying \( f(\theta_0) > 0 \). \( \theta_0 \) can be randomly drawn from a starting distribution, or simply chosen deterministically, for example, as the mean value.

2. For \( i = 1, 2, \ldots, n \):
   a. Sample a candidate \( \theta' \) from a jumping distribution or transition kernel \( f(\theta'|\theta_{i-1}) \), which is the probability of returning a value of \( \theta' \) given a previous value of \( \theta_{i-1} \). Theoretically, the jumping distribution could be any distribution. The only restriction in the Metropolis algorithm is that the jump density must be symmetric, i.e., \( f(\theta'|\theta_{i-1}) = f(\theta_{i-1}|\theta') \).
   b. Calculate the ratio of the densities
   \[
   r = \frac{f(\theta'|\theta_{i-1})}{f(\theta_{i-1}|\theta')}
   \]
   (11)
   where \( f(\theta|\theta) \) is calculated with Eq. (6), therefore,
   \[
   r = \frac{kL(\theta'|\theta_{i-1})}{L(\theta|\theta_{i-1})} \frac{L(\theta_{i-1}|\theta')}{L(\theta|\theta_{i-1})}
   \]
   (12)
   (c) Set \( \theta = \theta' \) with a probability of \( \min(r, 1) \); otherwise set \( \theta = \theta_{i-1} \), which means the jump is not accepted. This can be obtained as
   \[
   \begin{align*}
   \theta &= \theta' \quad \text{if} \ U \leq r \\
   \theta &= \theta_{i-1} \quad \text{if} \ U > r
   \end{align*}
   \]
   (13)
   where \( U \) is a random number from the uniform distribution of (0, 1).
   d. Stop the iteration if \( i = n \); otherwise \( i = i + 1 \) and go to Step (a).

A good jumping distribution should be chosen to make sure the jumps are not rejected too frequently; otherwise the random walk will waste much time standing still. The method to select a jumping distribution will be demonstrated in the illustrated examples.

3. Illustrative examples

An anchored rock slope with a joint, which was considered by Shukla and Hossain [34], is shown in Fig. 4. The slope has a height of 12 m and a slope angle of 60°. The joint starts from the toe and extends at a slope of 35°; it turns up vertically at an elevation of 7.65 m. The slope is reinforced with 4 rows of anchors 28 mm in diameter. Each anchor is pre-stressed at a force of 20 kN. The grouted part has a length of 4 m and a diameter of 120 mm. The parameters of the rocks and joint are shown in Table 2. The cohesion \( c \) and friction angle \( \phi \) of the joint are set as two random variables. \( c \) and \( \phi \) are assumed to follow lognormal distributions with means and standard deviations of 30 and 6 for \( c \), and 25 and 4 for \( \phi \). A uniform load \( P \) is applied on the crest of the slope and is assumed as a normal variable with a mean of 300 kN/m and a standard deviation of 30 kN/m. Three monitoring points are set to capture the safety state of the slope: the vertical displacement at point \( A \) \( (Y_A) \), the horizontal displacement at point \( B \) \( (X_B) \), and the horizontal stress at point \( C \) \( (S_C) \), as shown in Fig. 4.

A Bayesian network is constructed to integrate the monitoring information and evaluate the slope safety, as shown in Fig. 5. To illustrate the proposed method, only three uncertain slope parameters are involved; namely, the load \( P \), the cohesion \( c \) and the friction angle \( \phi \) of the joint. In this network, the change of any of the three parameters would result in changes of the factor of safety.

![Fig. 4. An anchored rock slope with a single slip surface (based on [34]).](image)

![Fig. 5. A Bayesian network that relates the factor of safety to monitoring parameters.](image)
safety ($F_S$) and the three monitoring items, namely $Y_a$, $X_b$ and $S_C$, as shown in Fig. 4. The slope safety ($S/F$) is related to $F_S$: the slope is safe if $F_S$ is larger than 1. A larger $Y_a$, $X_b$, or $S_C$ would indicate a more dangerous situation or a smaller $F_S$. This can be qualitatively explained within the Bayesian network: a larger $Y_a$ may be caused by a larger $P$ or a smaller $c$ or $\phi$; the larger load or the smaller shear strength parameter would lead to a smaller $F_S$. This phenomenon will be quantitatively demonstrated in the next sections.

3.2. Quantification of the Bayesian network

The quantification of the Bayesian network is to find the prior probabilities for the nodes, which include the probability distributions of the basic nodes (without parents) and the conditional probability distributions of the remaining nodes given their parents. The probability distributions of the basic nodes, namely the load ($P$), cohesion ($c$) and friction angle ($\phi$), are shown in Table 2. The conditional probability of $S/F$ given $F_S$ is easy to obtain, since it fully depends on $F_S$. Thus, a key task in this section is to quantify the conditional probability distributions for $F_S$, $Y_a$, $X_b$ and $S_C$ given the three basic nodes. This is realized using finite element analysis and a response surface method including the model errors.

GeoStudio [35] is a finite element software package, which consists of 8 components such as SIGMA/W for stress and displacement analysis and SLOPE/W for slope stability analysis. In this study, SIGMA/W is first applied to analyze the stress–strain behavior of the rock mass and the joint and the factor safety at a load ($P$) of 270 kN/m, a friction angle ($\phi$) of 17° and a cohesion ($c$) of 36 kPa. In this case, the calculated results are $Y_a = 15.2$ mm, $X_b = 14.9$ mm, $S_C = 112.4$ kPa and $F_S = 0.940$.

According to Eq. (4), the response function is given by

$$y = R(P, c, \phi) + e_1 + e_2$$

where $y = \mathbf{F}_S, Y_a, X_b$ and $S_C$; $x_1 = P, x_2 = c$ and $x_3 = \phi$; $R$ = the response functions of $y$; coefficients $\alpha_{10}, \alpha_{20}, \alpha_{11}$ and $\alpha_0$ are vectors obtained with the least squares method based on the numerical simulations as shown in Table 3. To check the accuracy of the response surface functions, 15 more points are randomly drawn from the space of $\theta$, and the $F_S, Y_a, X_b$ and $S_C$ values corresponding to these parameters are evaluated with both the finite element method and the third order polynomial function, as shown in Fig. 8. The results from the two approaches are close but not exactly the same. Hence, the response surface model error, $e_2$, is introduced to characterize the difference between the response surface method and the finite element method. The mean value of $e_2$ is zero. The standard deviations of $e_2$ can be estimated with statistical methods, as shown in

![Fig. 6](image-url)

**Fig. 6.** Finite element model for the anchored rock slope.

![Fig. 7](image-url)

**Fig. 7.** Results of finite element analysis: (a) strain contours; (b) slope stability profile.

**Table 3**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$Y_a$</th>
<th>$X_b$</th>
<th>$S_C$</th>
<th>$F_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{11}$</td>
<td>-0.0119</td>
<td>-0.0136</td>
<td>-0.0827</td>
<td>-0.0001</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>0.0146</td>
<td>0.0070</td>
<td>0.2291</td>
<td>0.0001</td>
</tr>
<tr>
<td>$x_{13}$</td>
<td>0.0102</td>
<td>0.0194</td>
<td>0.2090</td>
<td>0.0008</td>
</tr>
<tr>
<td>$x_{21}$</td>
<td>0.0790</td>
<td>0.0107</td>
<td>0.6887</td>
<td>0.0007</td>
</tr>
<tr>
<td>$x_{22}$</td>
<td>-0.1674</td>
<td>-0.1019</td>
<td>-0.7100</td>
<td>-0.0040</td>
</tr>
<tr>
<td>$x_{23}$</td>
<td>0.1507</td>
<td>0.1303</td>
<td>1.4936</td>
<td>0.0005</td>
</tr>
<tr>
<td>$x_{31}$</td>
<td>3.4278</td>
<td>3.7936</td>
<td>6.7262</td>
<td>-0.0036</td>
</tr>
<tr>
<td>$x_{32}$</td>
<td>-1.4040</td>
<td>-1.8430</td>
<td>-10.9293</td>
<td>0.0355</td>
</tr>
<tr>
<td>$x_{33}$</td>
<td>-2.3221</td>
<td>-2.5536</td>
<td>-24.1761</td>
<td>0.0776</td>
</tr>
<tr>
<td>$x_0$</td>
<td>15.5112</td>
<td>15.6990</td>
<td>76.3897</td>
<td>1.0434</td>
</tr>
<tr>
<td>$\sigma_{12}$</td>
<td>1.0400</td>
<td>1.2803</td>
<td>8.5492</td>
<td>0.0225</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9712</td>
<td>0.9656</td>
<td>0.9482</td>
<td>0.9679</td>
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</tbody>
</table>
from slope stability analysis can be assumed as a zero-mean normal distribution \( N(0, C) \), in which \( h(\cdot) \) could be any function and \( C \) is the covariance matrix. In this case, the distributions of both \( g(0^* - \theta_{-1}) \) and \( g(0^* - 0^-) \) are identical and follow \( N(0, C) \). Let us set \( g(0^* - \theta_{-1}) \) as \( 0^* - \theta_{-1} \). Thus, \( 0^* \) follows a normal distribution, \( N(0_{-1}, C) \). Zhang et al. [32] suggested that good calculation efficiency can be attained when \( C = 0.5C_p \), in which \( C_p \) is the covariance matrix of the prior distribution of \( \theta \).

In this study, the jumping distribution has a constant covariance matrix but a varying mean vector, since \( \theta_{-1} \) changes along the Markov chain. According to the procedure of MCMC simulation, the distributions of \( P, c \) and \( \phi \) are updated. Fig. 10 shows the updated \( P, c \) and \( \phi \) with the monitoring values of \( Y_A = 15 \text{ mm}, X_B = 15 \text{ mm} \) and \( S_C = 80 \text{ kPa} \). The samples for MCMC move randomly and achieve stationary states efficiently. The updated means and standard deviations of \( P, c \) and \( \phi \) are 296.52 and 23.57 kN/m, 29.92 and 5.81 kPa, and 24.80\(^\circ\) and 3.04\(^\circ\); respectively. Note the prior distributions of \( P, c \) and \( \phi \) are \( N(300, 30), \operatorname{LogN}(30, 6) \) and \( \operatorname{LogN}(25, 4) \). The standard deviations of the updated parameters are smaller than those of the priors, hence the uncertainties are reduced when the monitoring information is included. With the updated \( P, c \) and \( \phi \), the updated factor of safety is easily obtained according to Eq. (14). In this case, the updated mean and standard deviation of \( F_S \) are 1.06 and 0.08; the corresponding failure probability is \( P_F = 0.227 \).

### 3.4. Sensitivity of slope safety to monitoring information

Generally, the slope would be more dangerous when \( Y_A, X_B \) and \( S_C \) become larger. As shown in Fig. 11, \( F_S \) decreases and \( P_F \) increases with an increase of any of these three monitoring parameters. This is because that larger displacements or stresses reflect a larger load or a smaller cohesion or friction angle of the joint, which then leads to a smaller \( F_S \) and a larger \( P_F \). The curves of \( Y_A \) and \( X_B \) are quite symmetrical to keep calculations efficient, i.e. \( f(0^*|0_{-1}) = f(0^-|0_{-1}) \).

#### 3.3. Evaluation of slope safety with monitoring information

MCMC simulation is used to update the slope parameters (i.e. \( P, c \) and \( \phi \) in this study), \( F_S \) and \( P_F \) with the monitoring information. The jumping function \( f(0^*|0_{-1}) \) needs to be determined before conducting the MCMC simulation as introduced earlier. Here \( 0_{-1} \) is the current sampled vector of \( (P, c, \phi) \) and \( 0^* \) is the candidate vector of \( (P, c, \phi) \) generated from \( 0_{-1} \) with the jumping function. Theoretically, \( f(0^*|0_{-1}) \) could be an arbitrary distribution. However, according to the Metropolis algorithm [33], \( f(0^*|0_{-1}) \) must be chosen as

![Fig. 8. Comparison of results calculated using the response function (FS, SC, XB1, YA1) and the finite element method (FS2, SC2, XB2, YA2).](image)

![Fig. 9. Prior distributions of: (a) \( Y_A \); (b) \( X_B \); (c) \( S_C \) and (d) \( F_S \).](image)

![Fig. 10. Samples for MCMC simulation: (a) \( P \), (c) \( c \) and (e) \( \phi \); and the corresponding histograms (b) \( P \), (d) \( c \) and (f) \( \phi \).](image)

### Table 3. According to Tang et al. [36] and Zhang et al. [37], the model error, \( \varepsilon_k \), of \( F_S \) from slope stability analysis can be assumed as a biased normal distribution, \( N(-0.025, 0.054) \). Zhang and Chu [38] found that the estimated displacements of soil and rock in pile engineering follow an unbiased normal distribution with a coefficient of variation (COV) of 0.2–0.3. Thus, in this study, each of the model errors of \( Y_A, X_B \), and \( S_C \) is assumed to follow a normal distribution, \( N(0, 0.25\mu) \), where \( \mu \) is the estimated mean value of \( Y_A, X_B \) or \( S_C \). Conducting Monte Carlo simulation (1 million samplings) with Eq. (14), the prior distributions of \( Y_A, X_B, S_C \) and \( F_S \) are obtained, as shown in Fig. 9. In this case, the mean and standard deviation of the prior \( F_S \) are 1.04 and 0.09. This only shows a general understanding of \( F_S \) without any monitoring information. The \( F_S \) can be updated with the monitoring information of \( Y_A, X_B \) or \( S_C \), which will be illustrated in the next section.
close, because the displacements at points A and B, either in the horizontal or in the vertical direction, are similarly influenced by the slope parameters. This does not suggest that one of these two monitoring items is redundant. As shown in Fig. 12, without the information of $Y_A$, $F_S$ decreases with $X_B$ rapidly. This is because $F_S$ only relies on $X_B$. With the information of both $Y_A$ and $X_B$, the updated slope parameters integrating these two monitoring values will lead to a gentler curve for $F_S$. As $Y_A$ increases from 12 to 24 mm, $F_S$ decreases obviously. When both the two monitoring indexes are large, we may have more confidence that the slope is more unsafe.

There are intersections between the curves with and without including the information of $Y_A$ in Fig. 12. For example, the $F_S$ with $Y_A = 18$ mm is smaller than that without the information of $Y_A$ when $X_B$ is relatively small, (e.g., $X_B < 20$ mm), and vice versa. That is because $Y_A = 18$ mm is a dangerous sign for the situation only with the information of a small $X_B$ value, e.g. $X_B < 20$ mm; while it may be a safe sign for the situation only with the information of a large $X_B$ value, e.g. $X_B > 20$ mm. The fusion of the two monitoring indexes can also be illustrated in a straight-forward way as shown in Fig. 13. Larger values of $Y_A$ and $X_B$ imply a more dangerous state.

An important feature of the present method using Bayesian networks is that several different types of monitoring information (i.e., displacement, stresses, etc.) can be integrated into the slope-safety evaluation. Fig. 14 shows the results of two types of monitoring information: displacements ($Y_A$ and $X_B$) and rock stress ($S_C$). Generally, the rock stress is more sensitive to the load than the resistance parameters of the rock joint. Therefore, a larger $S_C$ reflects a larger $P$ more than a smaller $c$ or $\phi$. The $F_S$ and $P_f$ values in Fig. 14 are less scattered than those in Fig. 13, which means the uncertainty is reduced with the information of $S_C$. With an increase of $S_C$, the slope becomes less safe. In this figure, the safest state is when $Y_A$ and $X_B$ are both 3 mm and $S_C$ is 30 kPa. In this case, the $F_S$ is 1.158 and the $P_f$ is 0.108. The most dangerous situation is when $Y_A$ and $X_B$ are both 30 mm and $S_C$ is 130 kPa; the corresponding $F_S$ is as small as 0.897 and $P_f$ as large as 0.861.

In addition to the monitoring information, other qualitative information like engineering judgment can also be taken into the slope-safety evaluation in the present method. For example, if an engineer believes the slope is more likely to be safe based on his/her experience, then the information of “safe” or “$F_S > 1$” can be taken into the Bayesian network, as shown in Fig. 15(a) and (b). Comparing to Fig. 14(c) and (d), with this engineering judgment, $F_S$ becomes larger, $P_f$ becomes smaller, and the uncertainties of $F_S$ and $P_f$ are further reduced. If there is a heavy rain and the engineer believes the slope is more likely to fail, then the information of “failure” or “$F_S < 1$” can also be inputted into the Bayesian network, as shown in Fig. 15(c) and (d).
and might lead to changes in the model errors of $X_B$ and $S_C$. The correlations among the model errors can be modeled with a modified Bayesian network as shown in Fig. 16. The model errors, $e_{X_B}$, $e_{B}$, and $e_C$, are influenced by the model parameters (i.e., $P$, $c$, and $\phi$), the chosen model and other factors (e.g., finite element mesh, boundary conditions, and unknown factors). Note that $Y_A$ and $X_B$ influence the model errors as each of the model errors is assumed to follow a normal distribution, $N(0, 0.25\mu)$, where $\mu$ is the deterministically estimated value of $Y_A$, $X_B$, or $S_C$. Based on the concepts of Bayesian networks [20], if the factors influencing the model errors remain unknown, $e_{X_B}$, $e_{B}$, and $e_C$ are correlated; if all the factors are known, however, $e_{X_B}$, $e_{B}$, and $e_C$ will become independent. Thus, $e_{X_B}$, $e_{B}$, and $e_C$ are independent given the load, the cohesion and friction angle of the soil, and the simulation model. This Bayesian network can be simplified as Fig. 5 by integrating $e_{X_B}$, $e_{B}$, and $e_C$ into $Y_A$, $X_B$, and $S_C$, respectively. In this study, the node “other factors” in Fig. 16 is not considered.

In practice, the model errors may be influenced by many factors and may not be modeled in a simple way as in Fig. 5. In this case, the correlations among the model errors can be described using a correlation matrix. Thus, the likelihood function in Eq. (6) is a joint distribution. For example, when only two monitor parameters, $X_B$ and $Y_A$, are considered, the likelihood function is given by

$$L(\theta | X_B = x_B, Y_A = y_A)$$

$$\propto \int f(g_{X_B}(\theta) + e_X = x_B, g_{Y_A}(\theta) + e_Y = y_A)$$

$$\propto \int f(e_X = x_B - g_{X_B}(\theta), e_Y = y_A - g_{Y_A}(\theta))$$

$$\propto \int f(e_X, e_Y | x_B - g_{X_B}(\theta), y_A - g_{Y_A}(\theta))$$

(16)

where $x_B$ and $y_A$ are monitored values and $\theta$ are model parameters. Fig. 17 shows the influence of the correlation coefficient between the model errors of $X_B$ and $Y_A$, $\rho(e_{X_B}, e_{Y_A})$, on $F_S$. As $\rho(e_{X_B}, e_{Y_A})$ changes from 0 to 0.7, the uncertainty of $F_S$ becomes smaller and smaller. This is because $X_B$ and $Y_A$ with a strong correlation cannot change as freely as those with a weak correlation, leading to a smaller uncertainty of $F_S$.

4. Discussions

4.1. Correlation among monitoring parameters

In practical cases, the monitoring parameters may be correlated with each other. The correlations among the monitoring parameters are considered in the Bayesian network in two types: the correlations among the deterministically estimated monitoring parameters and the correlations among the model errors of the monitoring parameters. This can be explained with a modified equation from Eq. (14):

$$y = R(P, c, \phi) + \varepsilon$$

where $y$ is a vector of the three monitoring parameters, $Y_A$, $X_B$, and $S_C$; $\varepsilon$ is the model errors of $y$, namely $e_1$ and $e_2$ in Eq. (14). The correlations among the model errors are limited to those from numerical analysis ($e_2$). The correlations among the response surface model errors ($e_2$) are not considered in this study since this type of model errors are generated from regression analyses of separate response data.

The monitoring parameters are firstly correlated through the response functions with the same model parameters, namely $P$, $c$ and $\phi$. For example, a large $Y_A$ implies a large $P$ or a small $c$ or $\phi$, which may in turn lead to a large $X_B$ or $S_C$. Besides, the monitoring parameters may also be correlated through the model errors. For example, a change in the model error of $Y_A$ may lead to changes in the model errors of $X_B$ and $S_C$. The correlations among the model errors can be modeled with a modified Bayesian network as shown in Fig. 16. The model errors, $e_{X_B}$, $e_{B}$ and $e_C$, are influenced by the model parameters (i.e., $P$, $c$, and $\phi$), the chosen model and other factors (e.g., finite element mesh, boundary conditions, and unknown factors). Note that $Y_A$, $X_B$, and $S_C$ influence the model errors as each of the model errors is assumed to follow a normal distribution, $N(0, 0.25\mu)$, where $\mu$ is the deterministically estimated value of $Y_A$, $X_B$, or $S_C$. Based on the concepts of Bayesian networks [20], if the factors influencing the model errors remain unknown, $e_{X_B}$, $e_{B}$, and $e_C$ are correlated; if all the factors are known, however, $e_{X_B}$, $e_{B}$, and $e_C$ will become independent. Thus, $e_{X_B}$, $e_{B}$, and $e_C$ are independent given the load, the cohesion and friction angle of the soil, and the simulation model. This Bayesian network can be simplified as Fig. 5 by integrating $e_{X_B}$, $e_{B}$, and $e_C$ into $Y_A$, $X_B$, and $S_C$, respectively. In this study, the node “other factors” in Fig. 16 is not considered.

In practice, the model errors may be influenced by many factors and may not be modeled in a simple way as in Fig. 5. In this case, the correlations among the model errors can be described using a correlation matrix. Thus, the likelihood function in Eq. (6) is a joint distribution. For example, when only two monitor parameters, $X_B$ and $Y_A$, are considered, the likelihood function is given by
4.2. Features of the present method

Compared to the existing studies, the proposed method of slope safety evaluation using multi-source monitoring information has the following features:

1. Multiple sources of monitoring information can be taken into account. Bayesian networks are a very powerful tool for the integration of multi-source monitoring information. Physical analysis of slope stability with either analytic analysis or numerical analysis can be involved to obtain the prior probabilities. Monitoring information, engineering judgment and statistical data can also be used to update the networks and improve the safety evaluation. Within this new method, the uncertainties can be largely reduced and the evaluation will be more reliable with the support of multiple sources of information.

2. The model uncertainties can be considered. Previous studies show that model errors cannot be ignored. The proposed method deals with the correlated model uncertainties within the Bayesian network using MCMC simulation. The Bayesian method has been proven to be a useful tool to estimate the model errors by comparing the monitoring records and the model estimates [30,35,36]. Note that the Bayesian method is a special case of a Bayesian network by setting the parameters as the parent random variables.

4.3. Limitations of the present method

The present work has several limitations that should be improved in the future:

1. An engineered slope may involve several failure surfaces and several failure mechanisms such as excavation stress relief, seepage, earthquakes and anchor corrosion. How to identify likely failure mechanisms from monitoring data and how to deal with the system reliability issues involved [39] are subjects of further research.

2. The time effect of monitoring information needs to be considered, which may be caused by creep, aging anchors, crack development and so on.

3. The spatial variation of soil parameters should be considered in practical applications. Stochastic finite element methods may be used to establish the response functions of the factor of safety and monitoring parameters considering the spatial variability of soils and rocks.

4. The correlations among the model errors of different monitoring parameters need to be further evaluated with recorded monitoring data in specific case studies. This may be realized by comparing simulated results with recorded data using the Bayesian method.

5. Conclusions

This paper presents a systematic method for slope safety evaluation utilizing multi-source monitoring information. The following conclusions can be drawn:

1. A Bayesian network is constructed by considering the inter-relationships among soil parameters, loads, monitoring information and the factor of safety. The prior probabilities are quantified with finite element analysis and a response surface method including model and parameter uncertainties.

2. Markov chain Monte Carlo simulation is used to update the Bayesian network with multi-source monitoring information. The probability distributions of the parameters in the Bayesian network can be any types of distributions.

3. Different sources of monitoring information can be taken into account in the slope safety evaluation in the proposed method, including multiple types of monitoring data, engineering judgment and statistical data. More reliable evaluation is obtained with the support of multiple sources of site-specific information.

4. The spatial variation of soil parameters and the correlations among different monitoring parameters in specific engineering cases should be considered in the future to make the slope safety evaluation method more robust.

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References


